would be freezing of a given species (x_i = constant), two species having a given ratio, or whatever other information or prejudices one has about linear relations among the x_i This approach could be extended to the use of inequalities by using a nonlinear optimization routine[3] to minimize the Gibbs free energy, subject to lineai equality or inequality constraints

References

[1] Bahn, G S, "Thermodynamic calculation of partly frozen flows," AIAA J 1, 1960–1961 (1963)

[2] White, W B, Johnson S M, and Dantzig, G B, "Chemical equilibrium in complex mixtures,' J Chem Phys 28, 751–755 (1958)

[3] Rosen, J B, "The gradient projection method for nonlinear programming Part I Linear constraints," J Soc Ind Appl Math 8, 181–217 (1960); a program in FAP language for this approach is available as SHARE No 1399

Comments on "Wing-Tail Interference as a Cause of 'Magnus' Effects on a Finned Missile"

EDWARD R BENTON*
National Center for Atmopsheric Research, Boulder, Colo

AND

ANDERS S PLATOU†
Ballistic Research Laboratories, Aberdeen Proving Ground, Md

SINCE the publication of Ref 1, the present authors have had several private communications about their differ ences concerning the magnus effects on a finned missile [2] We find that our views actually are not too far different, and we have been able to reach agreement on the major issues The following is being published in order that the picture of the aerodynamics of a rotating wing will be clarified

1) The angle of attack and the lift distribution on a rotating wing have spanwise variations that are dependent on the rotation helix angle $\omega r/U$

2) The integrated lift on the wing and the lift distribution can be obtained from

$$L = \int_{a}^{a+s0} qC_{L\alpha}\left(\delta - \frac{r\omega}{U}\right) C(r)dr$$

where

q = dynamic pressure
$C_{L\alpha}$ = stationary wing lift curve slope
δ = wing deflection angle
r = spanwise distance from the center of rotation
ω = rate of rotation of the wing
U = forward velocity of the wing
C = wing chord $C = C(r)$ which is dependent on the wing geometry
a = distance from the rotation centerline to root chord
s_0 = distance between root chord and tip chord

3) The exact integrated lift and lift distribution cannot be determined until the wing geometry is fixed When the wing is in a free-spin condition, the net rolling moment is

Received October 31, 1963

* Staff Scientist Member AIAA
† Research Aerospace Engineer, Supersonic Wind Tunnels

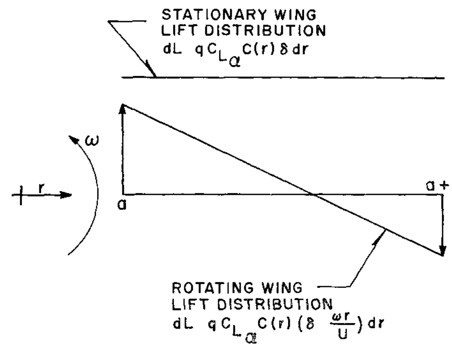

Fig 1 Comparison of the lift distribution on a stationary and a rotating wing

zero, $\int r dL = 0$, and the integrated lift can be shown to be small for any conventional wing (10 to 20% of the stationary lift) Also, the spanwise lift distribution can be approximated and is compared with the stationary wing lift distribution in Fig 1 If only the integrated lift is considered in the problem, then the rotation effect is small If the lift distribution along the span must be considered, then the rotation effects can be of prime importance

4) The change in the lift distribution due to rotation will alter the wake pattern aft of the wing and will change the wing-tail interference factors accordingly Also, the wing-tail interference factors are a function of the tail position in the wing wake The assumptions of $\eta_a = 0$ and $\eta_b = 1$ [Eqs (31) and (32) of Ref 2] apparently work well for the case considered but may not work for the general case

Refeiences

[1] Platou, A S, "Comments on Wing-tail interference as a cause of 'magnus' effects on a finned missile,' " AIAA J 1, 1963–1964 (1963)

[2] Benton, E R, ' Wing tail interference as a cause of 'magnus' effects on a finned missile, ' J Aerospace Sci 29, 1358–1367 (1962)

Comment on "A Theoretical Interaction Equation for the Buckling of Circular Shells under Axial Compression and External Pressure"

JOSEF SINGER*
Stanford University, Stanford, Calif

IN his note, Sharman[1] stated that he assumed $m = 1$ in the computations and reasoned that "assuming $m = 1$ restricts the valid solutions to positive external pressure only" The author believes that this reasoning is slightly erroneous, since it implies a jump in m at $(p/p_0) = 0$ Sharman's reasoning demands that, as $(p/p_0) \rightarrow 0$, m for minimum σ_c remains unity, until at $(p/p_0) = 0$, the well-known case of pure axial compression, $m \gg 1$ Hence at $(p/p_0) = 0$ there are two possible configurations, one with $m = 1$ and one with usual $m \gg 1$, which seems unlikely

For clarification, some points were calculated (with similar parameters as in Ref 1) near the R_c axis and the calculations indicated that there is indeed a narrow transition region, and

Received October 22, 1963

* Visiting Associate Professor, Department of Aeronautics and Astronautics, from Technion Israel Institute of Technology, Haifa, Israel Member AIAA

not a jump, at $(p/p_0) = 0$ For example with $\nu = 0\ 3$ and $(l/R) = 4$, one obtains the following:

1) For $(R/t) = 500$ at $(\sigma/\sigma_0) = 0\ 01$, the minimum φ_L occurs at $m = 3$ and not at $m = 1$

2) For $(R/t) = 200$ at $(\sigma/\sigma_0) = 0\ 02$, the minimum φ_L occurs at $m = 2$ and not at $m = 1$

Experimental evidence (although not directly applicable, since it refers to actual shells whose buckling behavior deviates from that predicted by linear theory) also confirms that $m > 1$ for some cases of combined axial compression and lateral pressure (see, for example, Ref 2)

References

[1] Sharman, P W, "A theoretical interaction equation for the buckling of circular shells under axial compression and external pressure," J Aerospace Sci **29**, 878–879 (1962)

[2] Weingarten, V I, Morgan, E J and Seide P, 'Final report on the development of design criteria for elastic stability of thin shell structures Space Technology Labs, Los Angeles, TR 60-0000 1945, p 175 (December 1960)

Reply by Author to J Singer

P W Sharman*

Loughborough College of Technology, Leicestershire, England

THE author is grateful to Dr Singer for pointing out the slight error in his statement

The main utility of an interaction equation is, of course, to provide a rapid means of design rather than an exact analysis An advantage of formulating such an equation in terms of "reserve factors" is that the denominators may be calculated from theoretical or empirical formulas, although such a mixture of theory and experiment is not really satisfactory However, in the absence of close agreement with theory and experiment, the procedure may be acceptable for design estimates

Received December 2, 1963

* Senior Lecturer, Department of Aeronautical Engineering

Comments on "Mach Number Independence of the Conical Shock Pressure Coefficient"

Wayne E Simon*

Martin Company, Denver, Colo

THE results presented in this note[1] may be compared with those of a recent technical note[2] which also presented an approximate equation for shock wave angle as a function of cone angle and Mach number In addition, the effect of specific heat ratio was included, and an equation for the surface pressure coefficient was developed Comparisons with exact results were shown for Mach number from 1 05 to 20 0, cone angle from 0 to the detachment value, and specific heat ratio from 1 1 to 1 8

Simpler equations were also presented [Eqs (12) and (13)][2] for the case when the sine of the cone angle was less than 85% of the detachment value In the nomenclature of the present note,[1] these equations would be

$$\sin\theta_w = \left[\frac{\gamma+1}{2}\sin^2\theta + \frac{1}{M_\infty^2}\right]^{1/2} \tag{1}$$

$$C_P = \left[\frac{\gamma+7}{4} - \left(\frac{\gamma-1}{4}\right)^2 + \frac{6}{M_\infty^6} + \frac{M_\infty^2-1}{M_\infty^4\sin\theta}\right]\sin^2\theta \tag{2}$$

Equation (10) of the present note, with θ in radians, is

$$\sin\theta_w = \left[\theta^{1\ 87} + \frac{1}{M_\infty^2}\right]^{1/2} \tag{3}$$

In the limit of Newtonian flow ($M_\infty \to \infty$, $\gamma \to 1$), Eq (3) becomes

$$\sin\theta = \theta_c^{0\ 935} \tag{4}$$

In the same limit, Eq (1) becomes

$$\sin\theta_w = \sin\theta \tag{5}$$

Since in Newtonian flow $\theta_w = \theta$, it is clear that the form of the approximation in the present note is not valid in the limit, even though the numerical values for air are reasonable In addition, the effect of specific heat ratio on shock angle is not considered

References

[1] Zumwalt, G W and Tang, H H, Mach number independence of the conical shock pressure coefficient," AIAA J **1**, 2389–2391 (1963)

[2] Simon, W E and Walter, L A, "Approximations for supersonic flow over cones,' AIAA J **1**, 1696–1697 (1963)

Received November 4, 1963

* Associate Research Scientist, Aerospace Division of Martin-Marietta Corporation Member AIAA

Reply by Author to W E Simon

Glen W Zumwalt*

Oklahoma State University, Stillwater, Okla

THE technical note of Simon and Walter[1] was published just after the final form of the paper being discussed[2] was submitted, so there was no opportunity for prepublication comparison of the two works As Simon points out, they have indeed succeeded in including the effect of the specific heat ratio in their conical shock approximations, and their curves then draw attention to the insensitivity of conical shock angle and surface pressure coefficient to γ values for the range applicable to perfect gas analysis They have obtained very good agreement for the $\gamma = 1\ 405$ value and for other γ values at the one cone angle of 20° No doubt they experienced the same difficulty as we in checking results for other γ values because of the lack of available published cone-flow solutions

In answer to Simon's criticism, it should be pointed out that our principal purpose was to call attention to the conical shock-wave pressure coefficient's strange behavior Secondarily, as a suggested use of this fact, an approximation for the conical wave angle was developed The resulting equation was similar in form to that of Simon and Walter, and it is not obvious to me that their equation is more simple Our equation fails to agree with the Newtonian limit, it is true However, Simon and Walter's equation (13) fails in exactly the same way, giving values almost identical to ours, unless the γ value is arbitrarily changed to unity The

Received December 4, 1963

* Associate Professor of Mechanical and Aerospace Engineering Member AIAA